**Chapter 04**

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## 4.1 Higher Order Linear Differential Equation

A linear differential equation of order is defined to be an equation of the form

For 1st order linear differential equations, we previous found

If , (i.e. the right-hand side of the equation is ,

is know as the complementary function.

is known as the particular integral.

The particular integral depends on the right-hand side of the equation.

For higher order linear differential equations, if , the equation is called a higher order linear homogenous differential equation with variable co-efficients.

## 4.2 Higher Order Linear Homogenous Differential Equations with Constant Co-Efficient

General Form:

Solution:

Say .

Let be a solution.

may also be a solution.

Let also be a solution.

may also be a solution.

is also a solution.

This is known as the **Principle of Superposition**. If to are the solutions of an th order linear homogenous differential equation with constant co-efficient, the complementary function and general solution is

can also be considered a solution.

(since cannot be )

This is called an auxiliary or characteristic equation. It will give two values of , and .

Generally,

The auxiliary equation is

is the general solution.

Let be a solution.

For auxiliary equations with distinct roots, values of will be found.

Let be a solution.

For auxiliary equations with repeated roots,

Say,

The auxiliary equation is,

For repeated roots,

is a factor.

For auxiliary equations with imaginary roots,

Let and .

is a solution.

## 4.3 Higher Order Linear Non-Homogenous Differential Equations with Constant Coefficients

For ,

We know, .

### Method of Undetermined Coefficient

This method helps us find when is polynomial, exponential, certain types of trigonometric or a linear combination of these three. It is complete guess-work.

|  |  |  |
| --- | --- | --- |
| U.C. Function | U.C. Set |  |
|  |  |  |
|  |  |  |
| or |  |  |
| or |  |  |

Solving Method:

* Solve corresponding homogenous differential equation ()
* Find the undetermined coefficient function and set
* Check repetitions between U.C. set and . If there are repetitions, revise U.C. set by multiplying the set containing repetitions by .
* Find the particular integral ()
* Plug and its required derivatives into the differential equation
* Equate the coefficients

U.C. Function:

U.C. Set:

Corresponding homogenous differential equation:

Auxiliary Equation:

U.C. Function

U.C. Set

Corresponding homogenous differential equation:

Auxiliary Equation:

U.C. Function ,

U.C. Set

, and are present in . and must be revised.

Revised U.C. Set

Corresponding homogenous differential equation:

Auxiliary Equation:

U.C. Function ,

U.C. Set

Corresponding homogenous differential equation:

Auxiliary Equation:

U.C. Function ,

U.C. Set

has a repetition, so the revised set for is

## 4.5 Higher Order Linear Differential Equations with Variable Coefficients

These are a special type of equation known as Cauchy-Euler Equations or Equidimensional Equations. The general form is,

In order to solve this, the equation must first be converted to a higher order linear differential equation with constant coefficients.

For example, for the equation

Let .

Also, , and .

Similarly,

Let .

Also, and

Thus, and

Corresponding homogenous differential equation:

Auxiliary Equation:

U.C. Function

U.C. Set

has a repetition and must be revised.

Let

and

Here, and .

Corresponding homogenous equation:

Auxiliary equation:

U.C. Function:

U.C. Set: